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ABSTRACT

A study examined the effects of manipulating the representation and organizational features of mathematical problems on students' abilities to solve them. Subjects for the study were 238 high school students who had demonstrated ability in computing percentages in a nonproblem solving format. The data analyzed in the study were student responses to test items varying in their representational (diagrammatic versus verbal) and organizational (pertinent versus extraneous) features. It was found that students' problem solving success was not significantly affected by representing percentage problems diagrammatically, but was powerfully affected by the presence of extraneous organizational features in the test items. The subjects' scores on those items that contained pertinent information only were significantly higher than those scores on items that also contained extraneous information. There was no specific hierarchy of difficulty found on the basis of representation and organization of features. (RL)

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Effects of Representation and Organizational Features of Text on Success in Mathematical Problem-Solving

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Effects of Representation and Organizational Features of Text on Difficulty in Mathematical Problem-Solving

Developing the ability to recognize, make inferences, and solve problems is an important goal at all levels of formalized schooling--from kindergarten through graduate school. Associated with meeting this objective are the tasks of identifying students' knowledge of skills related to problem-solving and determining effective instructional means for developing these abilities. For problem-solving in mathematics, Rasch(1960) proposes a simple model for describing the interaction which occurs between a student and a problem in mathematics: (1) the computational ability of the student and (2) the difficulty of the problem. These two factors are incorporated algebraically into a probability model that operates on the principle that the more able the student is, the greater his or her chance of success will be; and the more difficult the problem is, the smaller the chance will be that the student will solve it (Wright, 1977).

Some anecdotal evidence exists which proposes that representing math verbal problems diagramatcially may reduce the difficulty the student experiences in processing the necessary information to appropriately solve those problems. Pauk (1974), Thomas and Robinson (1977), Sherbourne (1977), and Singer and Donlan (1980) have suggested that by drawing an analytic data diagram or picture of the verbal problem students can visualize and clarify facts, principles, and relationships that are less evident from the words alone. In at least one study, some empirical evidence exists that making a diagram to illustrate pertinent information in a verbal problem significantly improves students' ability to solve the

problem (Sherrill, 1973). But, in all these cases, the diagrammatics were student-generated and used as aids to help the student understand the verbal problem. No studies were found where the authors of the verbal problems presented the pertinent information diagrammatically. What would the effect on student achievement be if pertinent information in mathematical problems were presented in diagrammatical form? It may be that when students are able to carry out the necessary computational procedures, presenting "verbal" problems in diagrammatical representation will decrease the difficulty of the problems and increase the probability that students will solve them correctly.

More is known about students' sensitivity to the organizational features of verbal material. Bruning (1970) studied pupils' factual recall for passages with both pertinent and extraneous information. He found that recall was significantly greater for passages which featured only pertinent information. A series of studies (Brown & Smiley, 1977; Brown, Smiley, Day, Townsend, & Lawton, 1977; and Brown & Smiley, 1978) has focussed on students' ability to extract the major points from verbal passages. Results indicate that the more mature processor is able to attend to the pertinent, or most informative material while the less able student finds difficulty in ignoring extraneous, or less informative sections of the passages. The practical aspect of these findings for sucess in problem-solving is evident. Effective channeling of pertinent information and sensitivity to extraneous details in verbal problems can insure successful problem-solving, again assuming that the students have acquired the necessary computational ability for those problems (Arter & Clinton, 1974; Biegen, 1972; Blakenship & Lovitt, 1976; and Faford, 1977). To date, we are not aware of any prior studies where the combined effects of representing verbal problems diagrammatically and featuring pertinent

and/or extraneous information on problem-solving ability was investigated.

In general, the purpose of this study was to begin examining the effects of manipulating the representation and organizational features of the problems on the students' ability to solve them. Our hope was that this and future studies will lead to instructional protocals for improving students' problem-solving performance. Specifically, this study examined students' ability to solve mathematical percentage problems when they were presented either diagrammatically or verbally including pertinent-only or pertinent and extraneous information. Once the students who demonstrated ability to compute percentage problems were identified, the following questions were investigated. Would students solve percentage problems represented diagrammatically more successfully than the identical problems represented verbally? Would these students solve percentage problems featuring pertinent-only information more successfully than those featuring both pertinent and extraneous information? And finally, would the combination of representation and feature result in a hierarchy of difficulty for solving percentage problems?

Method

Subjects

Two hundred thirty-eight 10th through 12th-grade students enrolled in an Algebra I or II course at five public high schools in Lafayette, Louisiana served as subjects for the study. These subjects were identified from a population of 670 by having demonstrated 80% or better on a screening test of percentage computational ability in a non-problem-solving format. Algebra I and II students were used as subjects in the study because, typically, percentages are assured to be mastered prior to entry into these courses and because problem-solving is usually stressed

in Algebra.

Instruments

Data analyzed in this study were from student responses to items involving percentages on two instruments--a computational screening test and a problem-solving test. Percentage problems were selected for use in the study because the ability to apply knowledge of percentages is recognized by persons in a variety of occupations as an essential mathematical skill (Saunders, 1980). The computational instrument included 10 percentage items: one-half were the type "A% of B = ____" (one-step computation problems) and the other half were the type "B - (A% of B) = ____" (two-step problems). These items involved only percentages less than or equal to 100%. They included no mixed percentages (i.e., no percentage numerals such as 56½ or 13.5%), and all items called for whole numeral answers. The order of the two types of items on the instrument was randomly assigned. Space was provided below each item for the students to show all their computations.

The problem-solving instrument included 10 items--two each in the following representations with these organizational features: (1) diagrammatical, pertinent-only information; (2) diagrammatical, pertinent and extraneous information; (3) verbal, pertinent-only information; and (4) verbal, pertinent and extraneous information. Eight different problem-solving situations involving percentages, four of the type A% of B = ___ and four of the type B - (A% of B) = ____, were devised to provide the basis for the items on the instrument. These eight problem-solving situations are briefly described below in Figure 1.

Insert Figure 1 here

Each situation was then stated in each of the four representations with the organizational features described above. Each representation and feature contained the same problem question (in prose presentation) appearing at the end of the verbal problems and at the bottom of the diagrammatical problems. For example, one problem-solving situation involved determining the interest earned on money placed into a savings account. As shown in Figure 2, this problem was constructed in diagrammatical and verbal representations with pertinent-only and pertinent-extraneous organizational features. Thus, from the original eight problem-solving situations devised, a total of 32 items were generated.

Insert Figure 2 here

These 32 items were randomly assigned to four forms of the instrument: each form contained 2 of each kind of representation and organizational feature (e.g., 2 diagrammatical, pertinent-only, etc.) but only one of the 8 problem-solving situations. A Latin Square design was used for randomizing the assignment and order of the items to each form to nullify any possible learning effects from one item to the next and to insure that no one of the four forms of the instrument would be more or less difficult than the other. In addition, each form contained 2 distractor items, 1 diagrammatical representation with pertinent-only features and 1 verbal representation with pertinent-only features. While each of the 32 regular items appeared only once across the four forms, the same 2 distractor items described above appeared on all four forms. Approximately four items appeared on each page of the four forms of the instrument with work space provided adjacent to each item for the students to show their computations. A cover-sheet for the instrument contained demographic

questions (i.e., name, school, grade-level, math course title, and sex) and instructions for completing the instrument. The reading difficulty level of the verbal representation items was computed to be between the 5th and 7th grade level, according to the Raygor Readability Estimator and the Fog Index Readability formulas.

Procedure

The instruments described above were administered to the original 732 subjects during a three-week period in September, 1980, by trained testers (graduate students in statistics). The testers replaced the classroom teachers during one regularly-scheduled 55-minute mathematics class period and administered both instruments. Students were not told before hand what type of mathematics problems they would see on the instruments. To prevent the possibility that the percentage computation test might prompt the students as to the types of mathematical computations they should use on the problem-solving test, the problem-solving instrument was administered first. Each student was randomly distributed one of the four forms of this instrument so that approximately one-fourth of the students in each classroom group completed each of the four forms. No time limits were invoked for this instrument, and all students completed it within approximately 25 minutes. The test administrators then collected the problem-solving instrument, distributed the computational screening instrument, and presented the directions as described above. All the students completed this instrument within approximately 15 minutes. For both instruments (though not directly applicable for the present study), students were expressly encouraged to show all of their jottings and mathematical computations for each item in the spaces provided.

For the purposes of this study, only the data from the problem-solving

instrument of those students who scored 80% or better on the computational screening instrument were analyzed. Data for items on the problem-solving instrument were bivariate: each item for each student was scored as either correct or incorrect. These data were divided for analysis according to the two types of percentage problems included on the instrument--Type I refers to "A% of B" one-step problems and Type II refers to "B - (A% of B)" two-step problems. The data for the two types of percentage problems were then analyzed using an analysis of variance for the following three variables: form (of the instruments), textual constraint (kind of representation and organizational feature), and item. Orthogonal contrasts were used to identify effects of representation (diagrammatical vs. verbal) and organizational feature (pertinent vs. extraneous information). Finally, a two-sample t-test was used to determine differences between the Type I and Type II percentage problems.

Results

The number of copies of the different forms of the problem-solving instrument available for analysis depended on the number of students taking each form who achieved 80 percent or better on the computational instrument. The totals are as follows: fifty-nine completed Form 1; fifty-two each completed Forms 2 and 3, respectively; and seventy-five completed Form 4. Results of the analysis of variance for forms, items and textual constraints for Type I percentage problems are presented in Table 1.

Insert Table 1 here

As shown in the table, the results indicate that, at a P-value of

.02, there was a significant difference between the textual constraints. Further analysis using orthogonal contrasts indicated that a highly significant difference ($p < .001$) existed between the organized features of pertinent-only and extraneous information. The mean number of correct responses for the pertinent-only features was 43.1 and 29.5 for the extraneous features. No significant differences ($p > .20$) were found between the diagrammatical and verbal representations. No significant differences ($p > .10$) were found to exist between the randomized ordering of the items on the four forms of the instrument. Although the items were randomly assigned to the four forms according to the Latin Square design to assure that no one form would be more or less difficult than the other, a significant difference ($p < .02$) between the forms for Type I items was found. This difference, however was due to the fact that more Form 4 instruments were included in the analysis than the other forms. The mean number of correct responses (of four items per form) to Type I percentage problems for the four forms were: Form 1--2.05; Form 2--2.60; Form 3--2.50; Form 4--2.60. (Note: The only analysis affected by varying number of the different forms used was the comparison of difficulty of the forms).

Insert Table here

A summary of the analysis of variance for forms, items and textual constraints for Type II percentage problems is presented in Table 2. Results of the analysis of variance indicate that, at a P-value of .04, there was a significant difference between the textual constraints. Again, further analysis using orthogonal contrasts showed a significant differences ($p < .02$) between the organizational features. The mean

number of correct responses for the pertinent-only feature items was 32.37 and the mean for the extraneous feature items was 28.0. Although the assignment of items to form and order of items presentation was randomized according to the Latin Square, significant differences were found between both the forms ($p < .002$) and the items ($p < .004$) for Type II items. As occurred for the Type 1 items, significant differences between forms were due to more Form 4 instruments having been completed than the others. The mean number of correct responses (of a total of four items) for each form was: Form 1--2.10; Form 2--2.08; Form 3--1.85; Form 4--2.01. (See previous note) Using Tukey's multiple-comparisons test, Item 5 (see Figure1) was determined to be significantly easier ($\bar{x}$ = 37.0) than the other "B - (A% of B)" items: $\bar{x}$ (Item 6) = 29.75, $\bar{x}$ (Item 7) = 28.25, and $\bar{x}$ (Item 8) = 24.75.

The results of a two-sample t-test for identifying differences between the Type I ($\bar{x}$ = 36.5) and Type II ($\bar{x}$ = 29.9) percentage problems showed a t-score of 3.91. At a P-value of .005, Type II "B - (A% of B)" two-step percentage problems were found to be significantly more difficult than Type I "A% of B" one-step problems.

Discussion

In response to the first question posed in this study, the data indicate that students' problem-solving success was not significantly affected by representing percentage problems diagrammatically. Evidently, the students who are able to compute percentage problems would not benefit from dealing with diagrammatically represented problem-solving situations before they are exposed to verbal problem situations. At first we were somewhat surprised to find that presenting mathematical problems diagrammatically did not significantly decrease difficulty.

The subjects in our study, however, had already mastered the computational aspect of problem-solving. Prior support for the use of diagrammatical representation has been given for the marginally successful student; and the representation has been viewed as an aid to problem-solving ability. A subsequent analysis of those data which includes students whose mastery of the percentage computations was marginal (i.e., between 50% and 70% on the computation instrument) might indicate that diagrammatical representation of mathematical problems does decrease the difficulty of those problems.

Turning to a discussion of the second question, subjects' problem-solving ability was powerfully affected by the presence of extraneous organizational features in the items. The subjects' scores on those items which included pertinent-only information were significantly higher than those scores on items which also featured extraneous information. This finding supports the contention of Arter and Clinton(1974), Biegen (1972), Faford (1977) and others that success in problem-solving is dependent on effective channeling of pertinent features with sensitivity to extraneous information. We see this finding as an elaboration upon the work conducted by Brown and Smiley (1977) and others who have concentrated their efforts on anlyzing students' ability to extract meaningful information from non-mathematical prose. However, their studies to examine learners' progress from immature to mature processors of text prose provide promising procedures for studying inference-making and problem-solving ability. Further studies are necessary which help to identify mature learners' awareness of the skills necessary to make inferences and solve problems, and then to provide to both the mature and immature learner an effective schema for developing and applying

those crucial skills.

In response to the third question, we presently are not able to note a specific hierarchy of difficulty of problem-solving situations on the basis of representation and organization of features. In this initial investigation, we determined only generally that mathematical percentage problems which included extraneous features, whether represented diagrammatically or verbally, were more difficult to solve than those problems which contained only pertinent information, also whether represented diagrammatically or verbally. In terms of "type" of problem investigated, problems of the type "A% of B" were more successfully solved than those of the type "B - A% of B". These results support the research conducted by Rasch (1960) and Wright (1977) which showed that two-step mathematical problems are more difficult to solve than one-step problems. Thus, with these subjects, the hierarchy of difficulty was dependent upon the type of mathematical percentage problem along with the organization of the information, not upon the representation of the problem situation.

Investigation concerning students' ability to make inferences and solve problems under various mainpulations of representation and organizational features is just beginning. Further studies involving students at all levels of ability with a wider variety of representations and organizational features are necessary to close in on effective strategies for solving problems. This line of research has an exciting nature about it; it serves to combine the recent efforts of the cognitive psychologists and reading specialists to analyze prose structure and schema with the work done by the mathematics specialists to develop learners' problem-solving ability. The better we are able to determine what and how representation and organization of features influence the difficulty of mathematical

problem situations, the better able we will be to develop strategies which are effective for helping students apply them and improve their ability to solve problems.

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Table 1

Analysis of Variance of Type I

Percentage Problems

Source	SS	df	MS	F	P-value
Forms	850.19	3	283.97	7.99	< .02
Items	345.19	3	115.06	3.24	> .10
Textual Constraints	783.19	3	261.06	7.36	< .02
Contrast Representations (Diagrammatical vs. Verbal)	18.06	1	18.06	.51	> .20
Contrast Features (Pertinent vs. Extraneous)	742.56	1	742.56	20.93	< .001
Remaining	22.57	1	22.57	.64	> .20
Error	212.88	6	35.48		

Table 2

Analysis of Variance of Type II

Percentage Problems

Source	SS	df	MS	F	P-value
Forms	424.19	3	141.40	20.02	< .002
Items	318.69	3	106.23	15.04	< .004
Textual Constraints	107.69	3	35.90	5.08	< .04
Contrast Representations (Diagrammatical vs. Verbal)	7.56	1	7.56	1.07	> .10
Contrast Features (Pertinent vs. Extraneous)	95.06	1	95.06	13.46	< .02
Remaining	5.07	1	5.07	.72	> .20
Error	42.38	6	7.06		

Figure 1

Problem-Solving Situations for Percentage Problems

A% of B Problems:

Item number	Description
1	Determine the interest on money placed in a savings account
2	Determine the tax to be paid on a restaurant bill
3	Determine the amount of weight reduced on a diet
4	Determine the amount of cheese eaten by a mouse

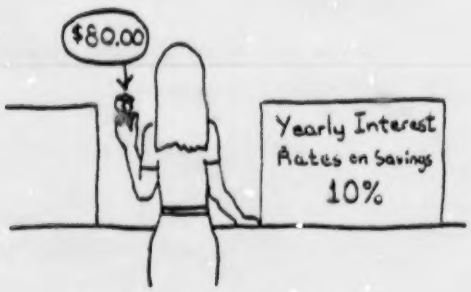
B - (A% of B) Problems:

5	Determine length of jeans after a percentage of shrinkage
6	Determine length of board after a percentage of cutting
7	Determine liters of gas remaining after percentage used
8	Determine depth of lake after percentage of drop in water level

Figure 2

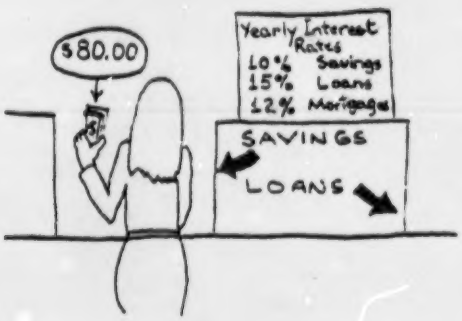
Example of Representation and Organizational Features

in a Problem-Solving Percentage Problem



How much money will Sandra earn in interest during the coming year?

Diagrammatical Representation with Pertinent-Only Organizational Features



How much money will Sandra earn in interest during the coming year?

Diagrammatical Representation with Pertinent and Extraneous Organizational Features

Sandra put $80.00 into her savings account. The yearly interest on savings accounts is 10%. How much money will Sandra earn in interest during the coming year?

Verbal Representation with Pertinent-Only Organizational Features

Sandra went to the bank this morning. She put $80.00 into her savings account. The yearly interest rate on savings accounts is 10%. The yearly rate on loans at the bank is 15%. Mortgages are going for 12%. How much money will Sandra earn in interest during the coming year?

Verbal Representation with Pertinent and Extraneous Organizational Features